crossings of this line with the lines l_3, l_4 and l_5 are S_3, S_4 and S_5. These points are centers of the circles C_3, C_4 and C_5 passing through the points F_∞ and A.

To obtain a measure for the reactance of the external circuit, we define

$$Q_1 = \frac{\omega L_1}{R_1 + R_K}, \tag{4}$$

the value of which is determined by the radii r_2 and r_5 as follows:

$$Q_1 = \frac{r_2}{r_5}. \tag{5}$$

For small values of Q_1 the point S_5 falls far outside the Smith Chart, and it is therefore more convenient to determine Q_1 from the angle α.

$$Q_1 = \operatorname{tg} \alpha. \tag{6}$$

The crossings of the circles C_3, C_4 and C_5 with the Q circle C_l are the points F_3, F_4 and F_5. The corresponding frequencies f_3, f_4 and f_5 are found on the frequency scale. The Q value of the unloaded resonator is then given by

$$Q_0 = \frac{f_5}{f_4 - f_2}. \tag{7}$$

The frequency f_5 is the resonant frequency of the unloaded resonator. Of course, Q_0 may be evaluated also from

$$Q_0 = (1 + \kappa) Q_L. \tag{8}$$

As it can be seen, the described method permits the evaluation of resonator parameters by a straightforward graphical analysis on the Smith Chart. There is, therefore, no need for auxiliary digrams, which are necessary in the other existing methods[2-4]

The limitation of the described method consists in the fact that it is usable only in cases where the losses in the coupling circuit can be represented by a series resistance only. It is possible, however, to modify the method also for such cases where the losses are represented by a parallel resistance only. The method becomes rather complicated if the losses are to be represented by a combination of series and parallel resistance.[5]

DARKO KAJFEŽ
Institute of Automation
Ljubljana, Yugoslavia

Calculating Coaxial Transmission-Line Step Capacitance*

As a frequent user of the curves given by J. R. Whinnery, H. W. Jamieson and T. E. Robbins,[1] I found it useful to arrive at a simple polynomial in powers of α and τ, which makes it possible to incorporate the step-capacitance calculation in a subroutine in a computer program dealing with transmission line calculations in coaxial lines.

From considering the peculiarities of the curves (Fig. 8, and Fig. 9 in the above mentioned article), the following form was chosen:

$$\begin{aligned} c'_d = {} & (a_1\tau^2 + b_1\tau + c_1)\alpha^2 \\ & + (a_2\tau^2 + b_2\tau + c_2)\alpha^1 + \cdots \\ & + (a_5\tau^2 + b_5\tau + c_5)\alpha^{-2} \text{ mpf/cm.} \end{aligned}$$

The coefficients for both cases (step on inner, step on outer) are given in Table I. These coefficients give a perfect fit at points $\alpha = 0.1$, 0.3, 0.5, 0.7, 0.9, and $\tau = 1$, 3, 5, and yield an accuracy of "line thickness" at any other point between the limits $0.1 \leq \alpha \leq 1.0$, $1 \leq \tau \leq 5$.

TABLE I

	Step on Inner		
	a_i	b_i	c_i
$i=1$	−0.771	+12 6	+ 58.2
2	+1.49	−23.8	−100.0
3	−0.778	+13.1	+ 27.7
4	+0.041 9	− 1.53	+ 15 5
5	+0.000 92	+ 0.069 9	− 0.726

	Step on Outer		
	a_i	b_i	c_i
$i=1$	−0 606	−4.100	+ 82 0
2	+1.13	+3.63	−138.6
3	−0 482	−1.36	+ 48.6
4	−0.115	+1.92	+ 10.2
5	+0.024 0	−0.182	− 0.397

P. I. SOMLO
Div. of Applied Physics
National Standards Lab.
C.S.I.R.O.
Australia

* Received June 5, 1963.

[1] J. R. Whinnery, H. W. Jamieson, and T. E. Robbins, "Coaxial line discontinuities," PROC. IRE, vol. 32, pp. 695–709; November, 1944.

Field Measurements Using Active Scatterers*

A general theory for analyzing scattering from loaded scatterers is available, and has been applied to small scatterers suitable for electromagnetic field measurements.[1] The theory is valid for both passive and active loads, as long as the load is linear. Ryerson has proposed the use of tunnel diodes to provide a negative resistance load, thereby enhancing the scattered signal.[2] His predictions have been verified experimentally by measurements on dipoles and tunnel diodes at S band.[3] The use of scatterers and tunnel diodes for field measurements is discussed in this communication.

The primary purpose of loading a small scatterer is to increase its echo area. The general formula is given by (18) of Harrington,[1] but in most cases the second term is large compared to the first term. Also, by reciprocity, the B of (18) is the scatterer gain times its input resistance; hence

$$\frac{\sigma}{\lambda^2} \approx \frac{1}{\pi} \left| \frac{G R_{in}}{Z_{in} + Z_L} \right|^2, \tag{1}$$

where σ = echo area, λ = wavelength, $Z_{in} = R_{in} + jX_{in}$ = the input impedance of the scatterer when used as an antenna, G is the directive gain of the scatterer when used as an antenna, and Z_L is the load impedance connected to the scatterer terminals. Note that the echo area is completely determined by the characteristics of the scatterer when used as an antenna. The extension of (1) to the case of bistatic scattering involves merely the replacement of G^2 by G_1G_2, where G_1 is the gain in the direction of the source and G_2 is the gain in the direction of the receiver.

By using a negative resistance load, one can make the denominator of (1) arbitrarily small, obtaining very large echo areas from small scatterers. In practice, the maximum echo area is limited by instabilities that arise. Some of the characteristics of small scatterers with negative resistance loads that are of importance in field measuring techniques are as follows. 1) The scatterer becomes extremely sensitive to proximity effects, because a small change in Z_{in} results in a large change in σ. Hence, such scatterers might be useful for the measurements in regions distant from objects, but probably not close to objects. 2) The scatterer behaves similarly to a resonant circuit with an effective quality factor

$$Q = \frac{|X_{in}|}{R_{in} + R_L}, \tag{2}$$

which becomes very large when R_L is negative. Hence, the scatterer becomes a very narrow-band device. This may be an advantage if a frequency-modulated system is used, as discussed in Sec. VIII of Harrington[1] 3) Because the scatterer behaves as a resonant circuit, it can be shown that a scatterer is characterized by a constant gain-bandwidth product, that is,

$$\sigma\beta^2 = \text{constant}, \tag{3}$$

where β = fractional frequency bandwidth between points where σ has fallen to 1/2 its value at resonance. Fig. 1 illustrates this

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[1] R. F. Harrington, "Small resonant scatterers and their use for field measurements," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 165–174, May, 1962.

[2] J. L. Ryerson, "Scatter echo area enhancement," PROC. IRE (*Correspondence*), vol. 50, p. 1979, September, 1962.

[3] J. Forgione, E. Calucci, and C. Blank, "The Application of Tunnel Diodes to a Reflecting Antenna Array," Applied Research Lab., Rome Air Development Center, Griffiss Air Force Base, N. Y., T.D. Rept. No. RADC-TDR-63-4; January, 1963.

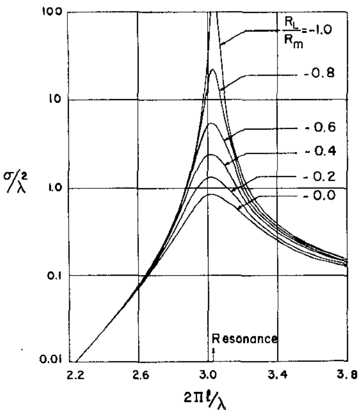

Fig. 1—Theoretical σ/λ^2 for a loaded dipole of length l, diameter $l/900$, center loaded by a negative resistance R_L.

behavior for a center loaded dipole. Because of (3), the bandwidth of a small loaded scatterer can be increased only by reducing σ, and vice versa.

Restricting the discussion now to tunnel-diode loads, one has a very effective way of modulating the scattered signal by modulating the bias on the diode. A voltage modulation of a few millivolts can swing a tunnel diode from the negative resistance region to the positive resistance region. The advantages of using a modulated scatterer for field measurements have been discussed in the literature.[4,5] If the bias were provided by a photoelectric cell, one could obtain the modulation by shining a modulated light on the cell. This would be similar to the technique of Vural, *et al.*,[6] but much more sensitive.

[4] J. H. Richmond, "A modulated scattering technique for measurement of field distributions," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp. 13–15; July, 1955.

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Roger F. Harrington
of Elec. Engrg. Dept.
Syracuse University
Syracuse, N. Y.

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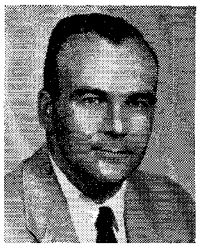
Contributors



Jack R. Baird (S'60–M'63) was born in Colchester, Ill., on May 1, 1931. He received the B.S. and M.S. degrees in electrical engineering in 1958 and 1959, respectively, from the University of Illinois, Urbana, where he is presently working on the Ph.D. degree.

From 1951 through 1954 he served in the U. S. Air Force as an Instructor in radar and radar test equipment at Lowry Air Force Base, Colo. He joined the Ultramicrowave Group at the University of Illinois in 1956 as an Undergraduate Technician designing and constructing high-power modulators, waveform generators, and special power supplies for low millimeter wavetubes. Since 1958 he has been actively engaged in research in the field of millimeter wave generation and detection. His research interests have included such problems as harmonic generation by field emission, frequency multiplication and mixing in a microwave discharge, megavolt electronics, and electron beam type frequency multipliers.

Mr. Baird is a member of Tau Beta Pi.

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Eric Brannen was born in Manchester, England, on September 25, 1921. He received the B.A. and M.A. degrees from the University of Toronto, Canada, in 1944 and 1946, respectively, and the Ph.D. degree from McGill University, Montreal, Canada, in 1948.

He served as Radar Officer in the RCNVR on loan to the Royal Navy in 1945. Since 1949 he has been a member of the Physics Department of the University of Western Ontario, London, Ont., Canada, where he is now Professor of Physics. His research work has been concerned with photon correlation in coherent light beams, nuclear decay schemes, electron dosimetry, the design and characteristics of the conventional and racetrack microtron electron accelerators, the interaction of high energy electron beams with materials, in particular, Cerenkov and transition radiation, applied to the generation of millimeter and submillimeter radiation, optical techniques in the submillimeter region and the interaction of submillimeter waves with various materials.

Dr. Brannen is a member of the American Physical Society and the Canadian Association of Physicists.

❖

Charles A. Burrus, Jr. was born in Shelby, N. C., on July 16, 1927. He received the B.S. degree, *cum laude*, from Davidson College, Davidson, N. C., in 1950, the M.S. degree from Emory University, Atlanta, Ga., in 1951, and the Ph.D. degree from Duke University, Durham, N. C., in 1955, all in physics.

At Duke University he studied under Texas Company and Shell Company Fellowships, and was employed at the University as Research Associate in 1954–1955. In 1955, he joined the Technical Staff of the Bell Telephone Laboratories, Holmdel, N. J. His work has been concerned primarily with microwave spectroscopy at millimeter and submillimeter wave frequencies, and with millimeter wave diodes.

Dr. Burrus is a member of Phi Beta Kappa, Sigma Pi Sigma, Sigma Xi, the American Physical Society and the American Association for the Advancement of Science.

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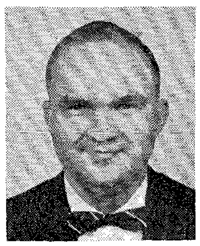

J. Clark Butterworth was born in Atlanta, Ga., on April 6, 1930. He received the Associate in Science degree from Southern Technical Institute, Chamblee, Ga., in 1951.

Since 1951, he has been on the staff of the Engineering Experiment Station, Georgia Institute of Technology, Atlanta, except for two years' service in the U. S. Army as